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Thermal analysis of a cryomicroscope: effect of heat spreading and contact resistance

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Abstract

The observation of crystal growth on a microscopic level is important to fields from metallurgy to cryobiology. The thermal behavior of a cryomicroscope stage has been analyzed by accounting for the spreading of heat and the thermal contact resistance between the solidification stage and the constant temperature plates. A scale analysis has been developed and a numerical solution has been obtained, and the results are presented and discussed. Temperature measurements have been used to estimate the thermal contact conductance. The results show that, although the thermal contact resistance cannot be ignored, the temperature gradient in the region of interest is uniform. Accounting for the thermal contact resistance and for the heat spreading is necessary to predict the order of magnitude of the temperature gradient over the metallic slide.

Nomenclature

Bi Biot number, $h\delta/k$
Bi_c contact Biot number, $h_c\delta/k$
c specific heat
d distance between reservoirs, Fig. 1
G temperature gradient
h heat transfer coefficient
h_c contact thermal conductance
H_f latent heat of fusion
k thermal conductivity
l length of each reservoir, Fig. 1
L length of the two reservoirs plus the gap between them, Fig. 1
Pe_d Peclet number based on the gap length d, $\rho Vcd/k$
Pe_L Peclet number based on the length L, $\rho VcL/k$
S shape factor
T temperature
V soldification rate
w slide width
x distance from the hot reservoir, Fig. 1
X dimensionless length, x/L
y direction normal to slide motion, Fig. 1
Y dimensionless length, y/δ.

Greek symbols
δ slide thickness, Fig. 1
θ dimensionless temperature, $(T-T_C)/(T_H-T_C)$
θ_∞ dimensionless temperature, $(T_\infty-T_C)/(T_H-T_C)$
ρ density.

Subscripts
C refers to cold reservoir
H refers to hot reservoir
∞ refers to ambient condition.

1. Introduction

The observation of crystal growth on a microscopic level is important to fields ranging from metallurgy to cryobiology. Cryomicroscopes, for example, have been used for over three decades to study the effects of cooling rate during the freezing process of biological materials and the subject has been reviewed [1]. Directional growth apparatuses for observing the solid–liquid interface have been used for many applications: crystal morphology [2–

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7], solute evolution [8], non-equilibrium effects [9], and many others.

Hunt et al. [10] have designed and built an apparatus that enables the control of the temperature gradient and of the solidification rate independently. Later, Rubinsky and Ikeda [11] presented an analysis of a similar apparatus with a description of the design issues (see also the work by Rubinsky et al. [12]). Among other constraints necessary to produce a constant temperature gradient over the observation region of the slide, Rubinsky et al. [12] neglected heat spreading and assumed that the thermal contact resistance between the moving slide and the constant temperature heat exchangers was negligible and described the temperature by

$$T(x) = T_{\mathrm{C}} - \left(\frac{x-l}{d}\right)(T_{\mathrm{H}} - T_{\mathrm{C}}), \quad l < x < l+d. \tag{1}$$

When contact resistance and heat spreading are considered, the temperature distribution is not given by equation (1), since the thermal gradient over the slide is significantly smaller.

Bianchi [7] has designed and built a directional growth apparatus based on the cryomicroscope developed to study the solid–liquid interface of transparent organic solutions [10] and the freezing of biological materials [11, 12], with a substrate made of aluminum and the constant temperature heat exchangers made of copper. The design is based on the directional solidification principle, using the Bridgman technique [13], with the advantage of controlling the temperature gradient and the solidification rate independently. Solidification of the phase change material occurs over an aluminum slide placed on two constant temperature heat exchangers. Even though the parts were carefully machined, the thermal contact resistance between the slide and the heat exchangers was never negligible and needs to be taken into account in evaluating the temperature gradient over the slide.

2. Mathematical model

Figure 1 shows a schematic diagram of the system considered and the temperature distributions over the aluminum slide. The sample freezes from the right to the left due to the lower temperature of the right heat exchanger. The aluminum plate slides over the two heat exchangers from the left to the right and the motion is imposed by a DC motor with controlled velocity V. After an initial transient, the aluminum slide moves to the right, but the interface region remains stationary with respect to the microscope.

The thickness of the sample and of the glass cover is much smaller than that of the metallic slide, so the temperature gradient in the sample is the same as that of the slide. Also, heat conduction through the slide is much greater than the latent heat released by solidification.

The problem is two-dimensional since the horizontal

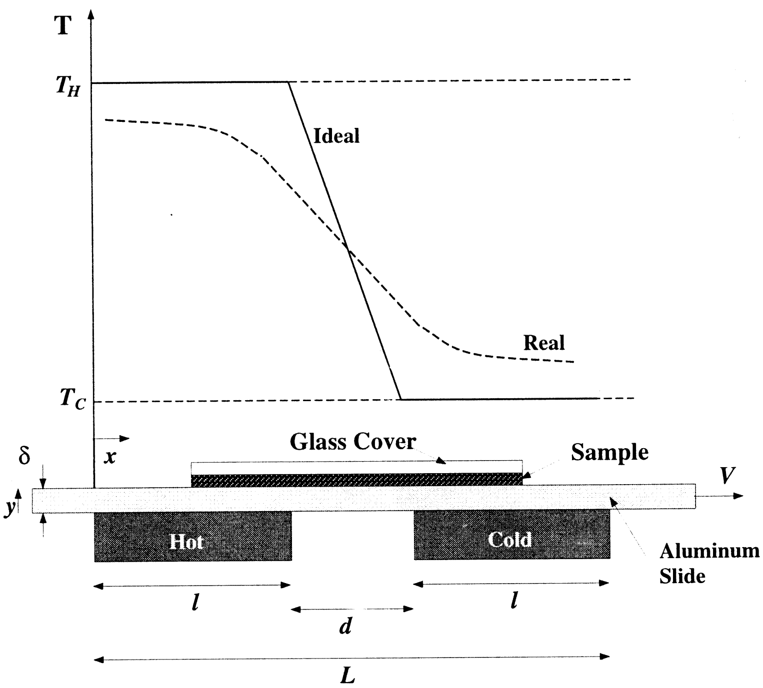

Fig. 1. Schematic diagram and temperature distribution over the aluminum slide.

heat conduction normal to the direction of the slide motion is negligible. The energy equation for the metallic slide is

$$\rho c V \frac{\partial T}{\partial x} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \tag{2}$$

The boundary conditions are:

$$\frac{\partial T}{\partial x} = 0 \quad \text{at}\, x = 0 \quad \text{and}\ x = L. \tag{3}$$

At $y = 0$ the boundary condition depends on the location along the slide: over the heat exchangers, there is heat conduction through the contact, and over the gap between the heat exchangers there is heat convection to the ambient air. At $y = \delta$, there is convective heat transfer to the ambient. So,

$$-k\frac{\partial T}{\partial y} = h_{c,\mathrm{H}}(T_\mathrm{H} - T) \quad \text{at}\, 0 < x < l, \quad y = 0 \tag{4}$$

$$-k\frac{\partial T}{\partial y} = h(T_\infty - T) \quad \text{at}\, l < x < l + d, \quad y = 0, \quad \text{and at}\, y = \delta \tag{5}$$

$$-k\frac{\partial T}{\partial y} = h_{c,\mathrm{C}}(T_\mathrm{C} - T) \quad \text{at}\, l + d < x < L, \quad y = 0. \tag{6}$$

By introducing the dimensionless variables X, Y, and θ, one can identify the dimensionless parameters that control the temperature gradient on the slide over the gap between the heat exchangers as Pe_d, Bi, Bi_c, θ_∞, d/L, δ/L.

3. Scale analysis

In order to impose a uniform temperature gradient over the gap d, conduction in the x-direction of the slide over the gap must be the dominant mode of heat transfer. It has to be greater than the convective losses (or gains) to the ambient and greater than the advective transport due to the motion of the slide.

The temperature throughout the thickness of the microslide has to be uniform. To satisfy this condition, convective heat transfer from the environment should be much smaller than conduction through the metal slide: $Bi \ll 1$. The order of magnitude of the Biot number for the apparatus is 1×10^{-5}.

Heat conduction through the slide has to be much greater than heat convection to the environment: $Bi\,(d/\delta)^2 \ll 1$. The order of magnitude of the parameter is 1×10^{-4}.

Heat conduction through the slide has to be much greater than heat advection due to the motion of the slide: $Pe_d = Pe_L(d/L) \ll 1$. The maximum value of the Peclet number based on the gap is 5×10^{-3}.

To assess the importance of the heat spreading and of the thermal contact resistance on the temperature gradient, one can assume that all of the above conditions have been satisfied. A thermal resistance network analysis was developed considering the contact resistance on both heat exchangers, the shape factor to change the direction of the heat flow, and the conduction thermal resistance, yielding a temperature gradient over the gap as

$$\frac{G}{(T_\mathrm{H} - T_\mathrm{C})/d} = \frac{1}{1 + 2(1/Bi_\mathrm{c})(\delta/d)(\delta/l) + 2(\delta/d)(w/S)} \tag{7}$$

where it was assumed that the thermal contact conductance was the same for both heat exchangers.

Thermal contact conductance of metals is relevant to many applications, but sliding contacts such as the one of interest in the present work have not received much attention [14]. Since the knowledge of the contact resistance between a sliding contact of copper and aluminum is not documented, the value of the contact Biot numbers cannot be directly calculated.

Separation of variables method can be used to solve the energy equation for a rectangular domain ($\delta \times l$) with isothermal boundary conditions to evaluate the shape factor S [15]. However, the solution leads to a divergent series. Rather than calculating S/w, which depends only on the geometry considered, the last term in the denominator of the right side of equation (7) is evaluated from the numerical solution.

For a perfect contact, the term involving Bi_c in equation (7) vanishes and the temperature gradient is the maximum, but the temperature distribution is not given by equation (1) because of the effect of the heat spreading. Analysis of equation (7) leads to the conclusion that poor thermal contact and heat spreading both prevent the temperature gradient to be equal to $(T_\mathrm{H} - T_\mathrm{C})/d$.

4. Results and discussion

The heat diffusion equation was solved using the control volume technique [16]. The metallic side is the region of interest, since it imposes its temperature to the sample being studied. A non-uniform grid 135×20 was used with a concentration of the control volumes over the gap between the two heat exchangers. The grid was chosen after tests to verify that the model predictions were grid independent, based on the value of the temperature gradient and the temperature distribution.

The standard values of the significant dimensionless parameters are based on the actual dimensions used in the experimental apparatus [7]: $Pe_d = 4 \times 10^3$, $Bi = 6 \times 10^{-5}$, $\theta_\infty = 0.7$, $d/L = 0.04$, $\delta/L = 0.024$. If the slide would have been made of glass, due to the smaller thermal conductivity, Pe_d would be 0.5 and Bi would be 10^{-2}. As the results will show, these values severely limit the use of such an apparatus and design changes would have to be made. The value of $Bi_{c,\mathrm{H}}$ and $Bi_{c,\mathrm{C}}$ were obtained from the solution by comparing the results obtained with the measured temperatures [7] and are found to be equal to $Bi_{c,H} = 1.8 \times 10^{-2}$ and $Bi_{c,C} = 9 \times 10^{-3}$.

4.1. *Effect of contact resistance*

The contact resistance is not the same over the two contacts as the measurements have shown [7]. However, to illustrate the influence of the magnitude of the contact resistance both contact Biot numbers were assumed to be equal. Figure 2 shows the temperature distribution at the slide surface for values of $Bi_c(d/\delta)(l/\delta)$ varying from 10^{-2} to ∞. For a very poor contact the temperature gradient in the gap is very small due to the large temperature drop across the contact. For $Bi_c(d/\delta)(l/\delta) = 0.3$, which is approximately its value for the experimental apparatus built, the temperature varies linearly over a distance even larger than the gap between the two heat exchangers. The temperature gradient is uniform (within 10%) over a region almost six times larger than the gap between the heat exchangers. The temperature distribution does not change for values greater than 10^3.

In order to use equation (7), S/w has to be determined. For a perfect contact ($Bi_c = \infty$), no slide motion ($Pe_d = 0$), and absence of convective losses ($Bi = 0$), the temperature distribution over the slide is also shown in Fig. 2. Even with a perfect contact, the temperature gradient is only $G = (0.528 \pm 0.032)(T_H - T_C)/d$, which is significantly lower than the ideal design value of $(T_H - T_C)/d$. The difference is only due to heat spreading, which is a geometric effect. The calculated value is the maximum temperature gradient attainable for the geometry chosen. From this calculation, $2(\delta/d)(w/S)$ in equation (7) can be evaluated and it is found to be 0.894.

The effect of the contact resistance on the temperature gradient can be revealed better in Fig. 3, where the dimensionless temperature gradient is shown as a function of the parameter $Bi_c(\delta/l)(\delta/d)$. For perfect contact and for very poor contact, the scale analysis, equation (7) shows an excellent agreement with the numerical solution, but it overpredicts the solution when the contact resistance and the heat spreading are of the same order of magnitude. Note, however, that the scale analysis is expected

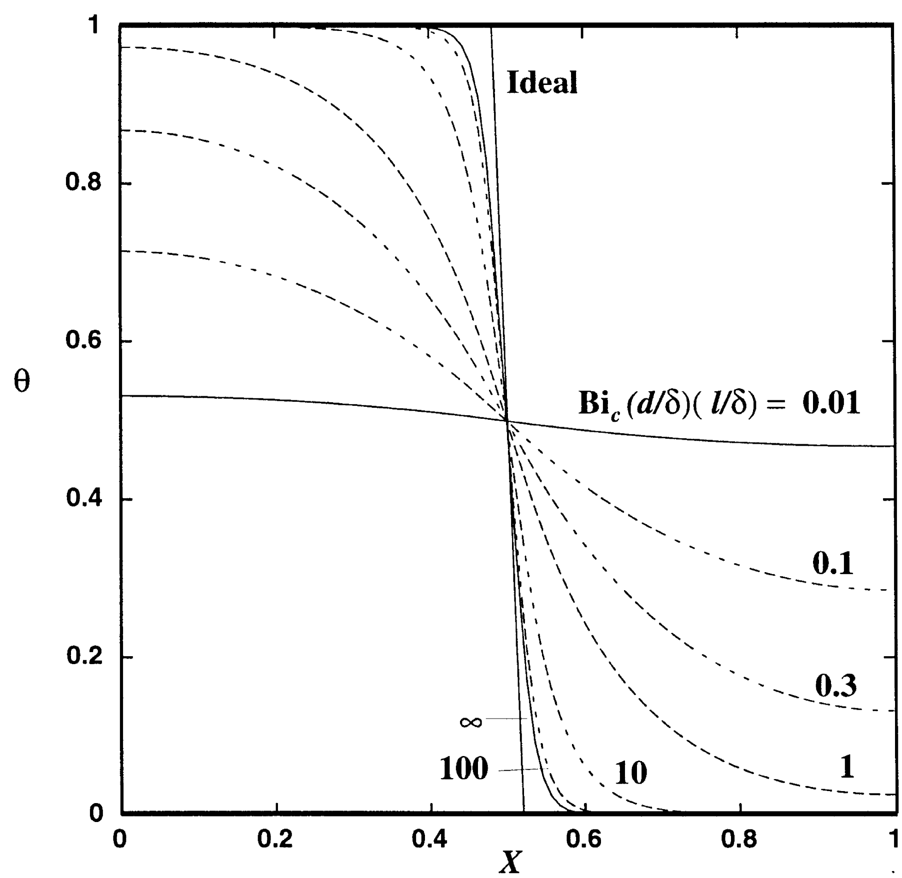

Fig. 2. Temperature distribution for various values of the contact Biot number and ideal temperature distribution.

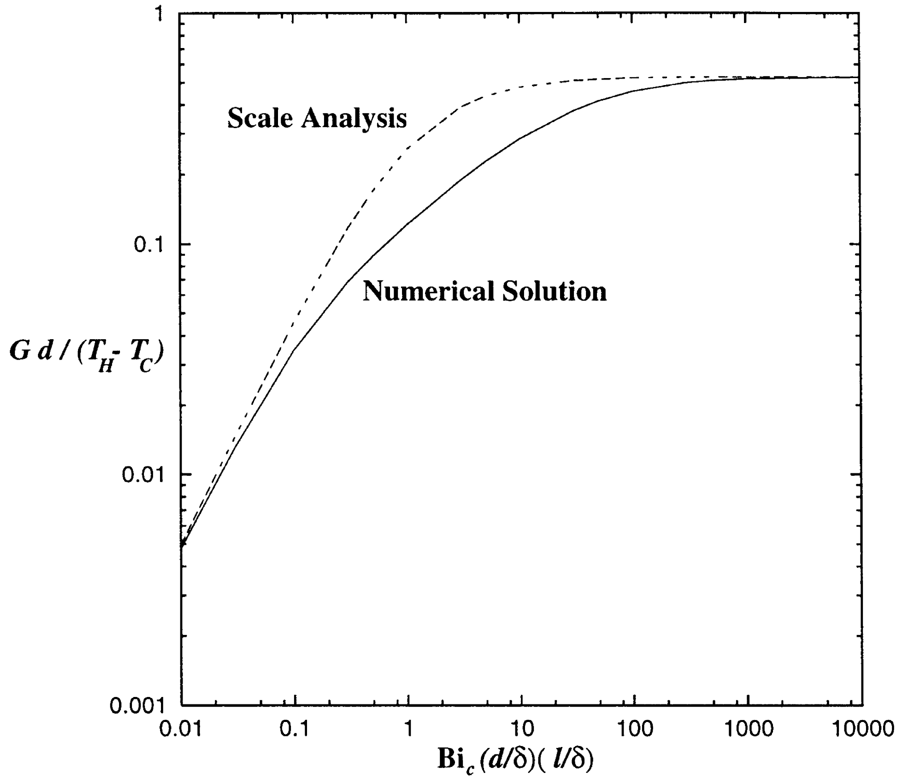

Fig. 3. Effect of the contact resistance on the temperature gradient.

only to predict the order of magnitude of the temperature gradient.

4.2. *Effect of slide velocity*

The effect of the slide velocity is shown in Fig. 4. For $Pe_d < 0.01$ the problem may be considered one of pure conduction, since advection is negligible. For slide velocities around 10 mm s^{-1} ($Pe_d = 1$), the temperature distribution is far from being governed by heat conduction. The curve generated for $Pe_d = 0.1$ suggests that the apparatus could be used for velocities up to 1 mm s^{-1}, but a correction would have to be made to accommodate the temperature shift due to advection.

4.3. *Effect of convection to the ambient*

Figure 5 shows the temperature distribution over the slide as a function $Bi(d/\delta)^2$. For values of this parameter smaller than 1×10^{-3}, convection to the ambient has no effect on the temperature distribution on the slide. Note that for a $Bi = 1 \times 10^{-1}$ the temperature distribution is greatly affected by convection to the ambient.

5. Conclusions

A parametric study was performed to analyze the heat conduction in a cryomicroscope. The energy conservation equation was derived by taking into account heat advection due to the slide motion, thermal contact resistance between the heat exchangers and the slide, and convection heat transfer to the ambient. A scale analysis was used to derive the conditions necessary to control the slide velocity and the temperature gradient on the slide over the gap between the heat exchangers independently. The energy conservation equation was solved numerically using the finite volume technique to evaluate the effect of the heat spreading, thermal contact resistance, heat convection to the ambient and slide motion on the temperature gradient. Temperature measurements from the experimental apparatus were used to estimate the thermal contact resistance. The following conclusion can be drawn from this study:

(1) Heat spreading is unavoidable and greatly reduces the temperature gradient on the slide over the gap. For the geometry chosen, even for a perfect thermal contact, the temperature gradient is only 53% of the idealized one.

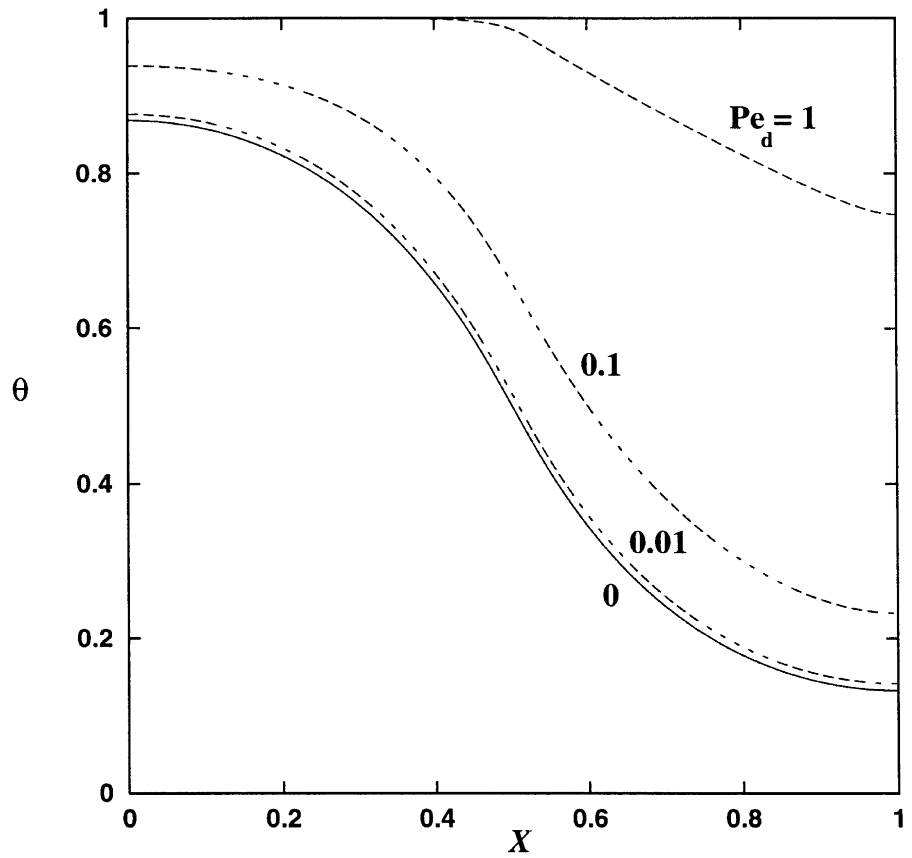

Fig. 4. Temperature distribution for various values of the Peclet number.

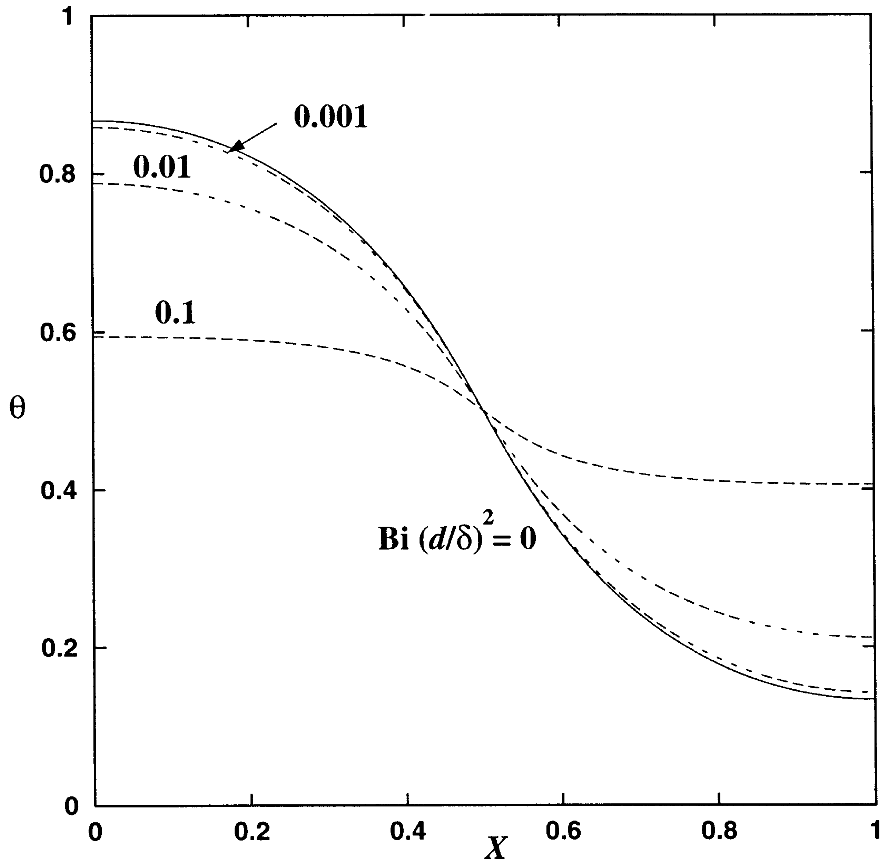

Fig. 5. Temperature distribution for various values of the convective Biot number.

(2) Thermal contact resistance is very important and needs to be considered when designing such a cryomicroscope. For the apparatus used in the present study, $Bi_c \approx 1 \times 10^{-2}$, which reduces the temperature gradient to about only 7% of the ideal design.

It is clear that research is needed to predict the thermal contact resistance between the moving slide and the heat exchangers. From the present study, it can be concluded that the assumptions of negligible heat spreading and negligible contact resistance can lead to a large overprediction of the temperature gradient on the slide over the gap between the heat exchangers.

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